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HYDRAULIC DRAG DUE TO DIVISION OF A STREAM OF FLUID INTO TWO PARALLEL CHANNELS WITH AN ARBITRARY RATIO OF FLOW RATES

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An expression is derived for the hydraulic drag and results of calculations are compared with experimental data.

The magnitude of the hydraulic drag at the inlet due to local separations of the stream during transfer of a liquid (or gas) from one channel to another can in many cases be found in [1]. As a rule, formulas recommended on the basis of experimental data are valid when the total flow rate remains constant during transfer from one channel to another. The theoretical solution obtained for straight channels of uniform cross-sectional area [2] and confirmed by results of experiments is valid only under that condition. There are semiempirical approximate relations available for determining the hydraulic losses which occur when a separate jet of fluid flows out of a stream (or into a stream) through a lateral channel at a given rate, at a given angle, and across a given area [1]. These formulas are, however, not sufficiently accurate for the simpler limiting cases such as, e.g., a zero exit angle or a zero flow rate through the lateral channel.

Here will be presented a theoretical solution to the problem, in the one-dimensional formulation, for determining the loss of total pressure due to entrance of a stream into a straight channel of uniform cross section from another one with a larger cross section. The smaller channel is completely inside the larger one and it takes up some arbitrary fraction of the total fluid flowing through the larger one (Fig. 1). A fluid here will include gases as well, but the effects of compressibility will be disregarded (Mach number $N_{Ma} \ll 1$). The hypothetical streamline along which the stream divides is indicated by dashes. The cross-sectional area of the stream, the pressure, the velocity, and the density of the fluid in channel 1 under steady state conditions (section 1-2) will be denoted as F_1, p_1, u_1, and ρ_1, respectively, and the corresponding parameters in channel 2 as F_2, p_2, u_2, ρ_2, respectively. In the segment of the initial stream in section 0−0 which subsequently enters channel 1 we denote the corresponding parameters as F_{01}, p_{01}, u_{01}, ρ_{01}; in the segment of the initial stream in this same section 0−0 which subsequently enters channel 2 we denote the corresponding parameters as F_{02}, p_{02}, u_{02}, ρ_{02}. The pressure, the velocity, and the density are assumed to be uniform within each thus defined segment of the stream cross section. The thickness of the layer between dividing stream segments is assumed to be zero,

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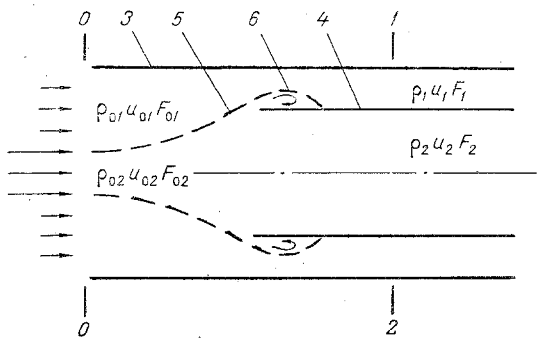

Fig. 1. Schematic diagram of the flow pattern: 0–0 and 1–2) reference sections; 3) wall of outer channel; 4) dividing wall; 5) dividing streamline; 6) zone of stream separation from the wall.

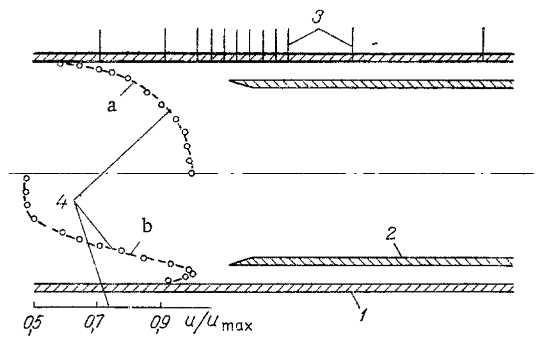

Fig. 2. Experimental model: 1) outer tube; 2) inner tube; 3) static-pressure gauges; 4) initial velocity profiles (a) and (b).

i.e., $F_1 + F_2 = F_{01} + F_{02} = F_0$.

The equations describing the conservation of flow rate and momentum in a straight channel of uniform cross section will be written as [3]

$$\rho_{01}u_{01}F_{01} = \rho_1 u_1 F_1, \quad \rho_{02}u_{02}F_{02} = \rho_2 u_2 F_2, \tag{1}$$

$$F_{01}(p_{01} + \rho_{01}u_{01}^2) + F_{02}(p_{02} + \rho_{02}u_{02}^2) = F_1(p_1 + \rho_1 u_1^2) + F_2(p_2 + \rho_2 u_2^2). \tag{2}$$

Friction at the walls has been disregarded here. We furthermore let $p_{01} = p_{02}$, $\rho_2 = \rho_{01}$, and $\rho_2 = \rho_{02}$ (heat transfer and mixing of streams will be disregarded). The loss of total pressure at the entrance to channels 1 and 2 will be sought in the form

$$\Delta p_1^* = p_{01} + \frac{\rho_{01}u_{01}^2}{2} - p_1 - \frac{\rho_1 u_1^2}{2}, \tag{3}$$

$$\Delta p_2^* = p_{02} + \frac{\rho_{02}u_{02}^2}{2} - p_2 - \frac{\rho_2 u_2^2}{2}. \tag{4}$$

From expressions (1)-(4) one can find the loss of total pressure in one channel, if the loss of total pressure in the other channel has already been determined on the basis of some particular considerations. When the dividing streamline is deflected toward channel 1 (as shown in Fig. 1), then one can assume that approximately $\Delta p_2^* = 0$ and thus simulate the flow in channel 2 without separation from the wall. After a few algebraic transformations, expressions (1)-(4) will then yield

$$\zeta_1 = \frac{2\Delta p_1^*}{\rho_1 u_1^2} = \left(1 - \frac{\bar{F}_1}{\bar{F}_{01}}\right)^2 \left(1 + \frac{\rho_{02} u_{02}^2}{\rho_{01} u_{01}^2} \frac{\bar{F}_1}{1 - \bar{F}_1}\right), \tag{5}$$

where $\bar{F}_1 = F_1/F_0$, $\bar{F}_{01} = F_{01}/F_0$. For $\bar{F}_{01} < \bar{F}_1$ this equation is ineffective.

When the dividing streamline is deflected in the direction opposite to that shown in Fig. 1, then expression (5) can be used for calculating Δp_2^* (on the assumption that $\Delta p_1^* = 0$) with subscripts 1 and 2 interchanged.

In order to calculate Δp_1^* according to expression (5) in the case of uniformly distributed parameters in the oncoming stream ($\rho_{01} = \rho_{02}$ and $u_{01} = u_{02}$), it is necessary to stipulate the relative area and gas flow rates in the channel under consideration: $\bar{F}_1$ and $\bar{G}_1 = \bar{F}_{01}$ ($\bar{G}_2 = 1 - \bar{G}_1 = \bar{F}_{02}$). When $\bar{G}_1 = 1$, then expression (5) yields the well-known theoretical solution [2]

$$\zeta_1 = 1 - \bar{F}_1. \tag{6}$$

The derivation of expression (5) has been based on a uniform distribution of gas velocity and density within each of the separating streams. This condition does not, as a rule, prevail in practice. Usually some sort of discontinuous change in these parameters occurs over the entire cross section of the not yet divided stream. In order to make expressions useful for calculating the hydraulic drag which occurs due to division of a stream with a beforehand known distribution of parameters over its cross section, it is necessary not only to stipulate $\bar{F}_1$ and $\bar{G}_1$ but also to calculate the mean-over-the-area velocity heads in zones 01 and 02. The magnitude of $\bar{F}_{01}$ needed for this can, moreover, be found from the solution to the equation

$$\overline{\rho u}_{01} \bar{F}_{01} = \bar{G}_1, \tag{7}$$

where $\overline{\rho u}_{01}$ is the mean-over-the-area mass rate of gas flow in zone 01, referred to its mean over the entire cross section of the initial stream.

Experiments were performed for the purpose of verifying relation (5), primarily in application to streams with a nonuniform velocity. The model in which the loss of pressure in an air stream was measured had been constructed in the form of a tube with an inside diameter of 34 mm around a coaxial with its other tube with an outside diameter of 28.2 mm and a wall thickness of 1.1 mm (Fig. 2). The spout of the inner tube was mounted as shown on the diagram (a 4-mm-deep chamfer rounded at the tip). The distribution of static pressure was measured along the outer wall of this annular channel formed by those two tubes with a sharp edge at the entrance. The location of static-pressure gauges connected to a V-tube differential water manometer is shown in Fig. 2. In the course of the experiment the rate of air flow G_0 and the air flow rate G_1 through the annular channel were also measured. Air was injected into this model at an ambient temperature of 290-295°K.

Three types of velocity profiles were set up in the initial stream (before division): one profile with the maximum velocity at the channel axis (profile *a* in Fig. 2), one profile with the maximum velocity near the wall (profile b in Fig. 2), and one profile with an almost uniform velocity (and a thin boundary layer at the wall). The first profile was produced by making the outer tube very long (1 m) beyond the inner tube, the other two profiles were produced by placing a bundle of meshes inside the outer tube at a distance of 100 mm before the inner tube. The velocity profiles were measured at a distance of 19 mm before the chamfer of the inner tube, where they already did not significantly depend on the varying ratio of flow rates in the separate streams.

The results of static-pressure measurements along the channel with velocity profiles *a* and b are shown in Fig. 3a, b, respectively, with the change in pressure from that in the section of velocity profile measurement plotted along the axis of ordinates, referred to the mean velocity head $q_1 = G_1^2/(2\rho_1 F_1^2)$ in the steady stream in channel 1 (the results of experiments with a uniform velocity profile at the entrance, obtained on the basis of results with profiles intermediate between profiles *a* and b, are not shown here). The relative air flow rate $\bar{G}_1$ in the annular channel served as the parameter for the pressure distributions. The point x = 0 corresponds to the beginning of the inner tube. Here the Reynolds number referred to the parameters of the initial stream in the outer tube was constant for each $\bar{G}_1 \neq 1$ and equal to $8.8 \cdot 10^4$ and for $\bar{G}_1 = 1$ was equal to $4.2 \cdot 10^4$. The absolute error of $\Delta p/q_1$ determinations did not exceed the 0.03 limit.

For a better presentation of the experimental data, the static pressure in the annular channel was calculated on the basis of the one-dimensional formulation of the problem (dash lines in Fig. 3). The magnitude of $\bar{F}_{01}$ needed for calculations according to expression (5) was found from the solution to Eq. (7) for the given velocity profiles (*a* and b) and the known magnitude of $\bar{G}_1$. The theoretical change in the static pressure due to contraction (or expansion) of jet tubes from F_{01} to F_1 before their entrance to the annular channel is depicted in Fig. 3 by a jump at x = 0. The hydraulic loss at the entrance calculated according to expression (5) (for the

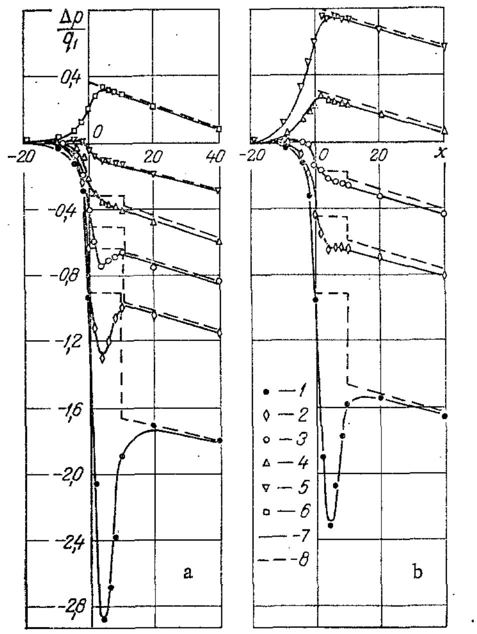

Fig. 3. Distribution of static pressure along the wall of the outer tube, x (mm) (*a* and b corresponding to velocity profiles *a* and b in Fig. 2, respectively): 1) $\bar{G}_1 = 1.00$; 2) 0.49; 3) 0.39; 4) 0.32; 5) 0.27; 6) 0.22; 7) approximation of experimental data; 8) calculation.

cases where $F_{01} > F_1$) is also depicted in Fig. 3, by a jump at x = 10 mm. A continuous variation of static pressure in this formulation of the problem can be determined only when the location of the dividing streamline and the shape of the separation zone at the channel entrance are given.

The coefficient of friction between the air stream and the wall of the annular channel, corresponding to the slope of the straight dash lines at $x \geq 10$ mm in Fig. 3, was experimentally determined from the drop of static pressure between x = 20 mm and x = 100 mm in the same annular channel. The static pressure was found to vary linearly over this distance. The friction coefficient was found to be ≈0.03 and, interestingly, almost independent of the Reynolds number referred to the stream parameters in the annular channel.

The data in Fig. 3 indicate an agreement between the results of calculations and measurement on the basis of the total change in static pressure in the stream upon entrance into the annular channel. A distinguishing feature of the experimental data (in contrast with the theoretical data presented here) is the appearance of an appreciable rarefaction along the x = 0–10-mm segment at high values of $\bar{G}_1$. It is caused by a contraction of the stream at the entrance to the annular channel due to its separation from the inner channel wall. Upon comparing the hydraulic entrance loss in the channel with the loss on shock behind the stream separation zone, one can estimate the maximum rarefaction on the basis of the Borda equation of a uniform stream.

In the case of isokinetic stream entrance to the annular channel, when the dividing streamline is a straight one parallel to the wall, there should be no hydraulic entrance loss. The straight dash line approximating the experimental data in the x = 10–40-mm segment should then almost pass through the origin of coordinates (its actual deviation from the origin of coordinates is due to an increase of friction at the chamfer of the inner tube). In the case of isokinetic stream entrance, moreover, calculations yield an $\bar{F}_{01}$ equal to 0.26 and 0.37 for velocity profiles *a* and b, respectively. We will note that $\bar{F}_1 = 0.31$ here. According to the experimental data, a perfectly isokinetic entrance of air to the annular channel was not realized in these tests. However, through the interpolation of the experimental data obtained at other values of $\bar{G}_1$ close to isokinetic ones, one can find the values of $\bar{G}_1$ for isokinetic entrance. These values of $\bar{G}_1$ were found to be close to the theoretical values 0.26 and 0.37, respectively.

It has been assumed in the calculations, just as in the derivation of expression (5), that there is no hydraulic loss at the channel entrance when $F_{01} < F_1$. It appears that when F_{01} differs appreciably from F_1 ($F_{01} < F_1$), the stream should separate from the wall of the outer tube opposite the chamfer of the inner tube and a hydraulic loss should occur as a consequence. Experiments with $\bar{F}_{01}$ up to 0.17 ($\bar{G}_1$ = 0.13 and 0.19 for velocity profiles *a* and b, respectively) were performed for the purpose of detecting such a loss. However, no significant difference in pressure was found between measurements and calculations based on the assumption of zero hydraulic entrance loss. This indicate that the loss of total pressure will be small when $\bar{F}_{01} = 0.17$ and $\bar{F}_1 = 0.31$, even with separation of the stream occurring.

The experimental data pertaining to small values of $\bar{G}_1$ did not yield an adequate basis for pinpointing the hydraulic loss (the measurements were very inaccurate). The measured increase of pressure in the annular channel at $G_1 = 0$ was equal to 0.46 and 1.22 times the mean velocity head in the oncoming stream with profile *a* and with profile b, respectively. No conclusions regarding separation of the stream can be drawn from these figures. Only the existence of some relation can be deduced from the fact that these figures correspond approximately to the ratio of the mean velocity head in a boundary layer of thickness equal to the height of the annular channel to the mean velocity head in the entire stream at section 0–0.

The measured pressure recovery in the inner tube at $\bar{G}_1 = 1$ was equal to the mean velocity head in the oncoming stream, a further confirmation of a small loss of total pressure in the inner tube with $F_{02} < F_2$. This reading in the inner tube and the agreement between theoretical and experimental data pertaining to the annular channel (Fig. 3) validate, to some extent, the earlier assumption that $\Delta p_2^* = 0$ in the derivation of the engineering formula (5).

NOTATION

ρ, density; u, velocity; p, static pressure; F, area; x, distance from the chamfer of the inner tube in the downstream direction; Δp, change of pressure along the channel from the pressure at the entrance section; q, velocity head; ζ, drag coefficient; subscripts; 1, outer channel; 2, inner channel; and 0, entrance section.

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